

## Weak Generalized Inverses and Ranks of Modified Matrices

by

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### Abstract

The problem considered is that of obtaining (weak) generalized inverses and ranks of matrices of the form  $\mathbf{R} + \mathbf{STU}$  (in terms of generalized inverses of  $\mathbf{R}$  and various other matrices or in terms of the ranks of these matrices). It is shown that generalized inverses of  $\mathbf{R} + \mathbf{STU}$  can be obtained as submatrices of generalized inverses of the partitioned matrix

$$\mathbf{C} = \begin{pmatrix} \mathbf{R} & -\mathbf{ST} \\ \mathbf{TU} & \mathbf{T} \end{pmatrix}$$

and that  $\text{rank}(\mathbf{R} + \mathbf{STU}) = \text{rank}(\mathbf{C}) - \text{rank}(\mathbf{T})$ . These results are used to translate various formulas for generalized inverses and ranks of partitioned matrices into formulas for generalized inverses of  $\mathbf{R} + \mathbf{STU}$  and for  $\text{rank}(\mathbf{R} + \mathbf{STU})$ . The formulas for generalized inverses of  $\mathbf{R} + \mathbf{STU}$  can be regarded as generalizations of Woodbury's formula (which is for an ordinary inverse).