

A COMPARISON OF THE COST-EFFICIENCIES OF THE SEQUENTIAL,  
GROUP-SEQUENTIAL, AND VARIABLE-SAMPLE-SIZE-SEQUENTIAL  
PROBABILITY RATIO TESTS

by

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ABSTRACT

Wald and Wolfowitz (1948) have shown that the Sequential Probability Ratio Test (SPRT) for deciding between two simple hypotheses is, under strong restrictions, optimal in three attractive senses. First, it can be a Bayes rule. Second, of all level  $\alpha$  tests having the same power, the test with the smallest expected number of observations is the SPRT, where this expectation is taken assuming either of the two hypotheses is true. Third, the level  $\alpha$  test needing the fewest expected number of observations is the SPRT, where this expectation is now taken with respect to both data and the prior over the two hypotheses. Principal among the strong restrictions is that sampling can proceed only in a one-at-a-time manner. This has left open the question of whether the SPRT's attractive properties persist in less restrictive environments. This paper shows, for almost all cases, that the answer is no and suggests that, under these circumstances, decision makers are better served by variable-sample-size and group-sequential decision rules.

KEY WORDS: Bayes optimality, error probabilities, prior, sequential decisions.