

**CONTINGENTLY AND VIRTUALLY BALANCED
INCOMPLETE BLOCK DESIGNS AND THEIR EFFICIENCIES
UNDER VARIOUS OPTIMALITY CRITERIA**

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Abstract: Even when a parameter set (v, k, λ) satisfies the necessary conditions for the existence of a Balanced Incomplete Block (BIB) design, the actual design may not exist or its existence may be unknown. We introduce two classes of designs, Contingently Balanced Incomplete Block (C-BIB) designs and Virtually Balanced Incomplete Block (V-BIB) designs, that may be considered in such cases. Both C-BIB and V-BIB designs are constructed from Unfinished Balanced Incomplete Block (U-BIB) designs, which can be constructed by a sequential search algorithm. Some V-BIB designs are shown to be highly efficient under A-, D-, and E-optimality criteria. Special attention is given to the parameter set $(22, 8, 4)$ for which the existence of a BIB design is unknown. Highly efficient V-BIB designs exist for this parameter set. Also, C-BIB and V-BIB designs for (v, k, λ) may be used to construct BIB designs for parameter sets $(v, k, t\lambda)$, where $t > 1$ is an integer. This generalizes the well-known result that multiple copies of a BIB design form again a BIB design.

Key words and phrases: A-optimality, BIB(22, 33, 12, 8, 4), BIB designs, Block designs, D-optimality, E-optimality, Optimal designs, simple random sampling.