

An Alternative Way to Establish the Necessity Parts of the Classical Results on the Distribution of Quadratic Forms

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Abstract

The proof of the necessity parts of the classical results on the statistical independence of quadratic forms or of second-degree polynomials (in normal variates) has been a long-standing source of frustration for those responsible for the teaching of courses in linear models. Those proofs that are currently available require knowledge of some relatively sophisticated mathematics and/or properties of cumulants and tend to be rather complicated. Consequently, they may be intimidating (and possibly inaccessible) to some students. In this paper, basic properties of polynomials in a single variable are used to construct an alternative proof. A similar method of proof can be used in establishing the necessity of the necessary and sufficient condition(s) for a quadratic form (or a second-degree polynomial) to have a noncentral chi-square distribution.

KEY WORDS: Quadratic forms; Statistical independence; Chi-square distribution; Multivariate normal distribution.