A Necessary and Sufficient Condition For Asymptotic Independence Of Discrete Fourier Transforms Under Short- and Long Range Dependence

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ABSTRACT

Let $\{X_t\}$ be a stationary time series and let $d_T(\lambda)$ denote the discrete Fourier transform (DFT) of $\{X_0, \ldots, X_{T-1}\}$ with a data-taper. The main result of the paper provides a characterization of asymptotic independence of the DFTs in terms of the distance between their arguments under both short- and long-range dependence of the process $\{X_n\}$. More precisely, it is shown that under some regularity conditions, $d_T(\lambda_{1T})$ and $d_T(\lambda_{2T})$ are asymptotically independent *if and only if* $|T(\lambda_{1T} - \lambda_{2T})| \to \infty$ as $T \to \infty$. Furthermore, asymptotic joint distributions of $d_T(\lambda)$'s are also established for the cases '$T(\lambda_{1T} - \lambda_{2T}) = o(1)$ as $T \to \infty$' (local ordinates) and '$|T(\lambda_{1_T} - \lambda_{2_T})| \to \infty$ as $T \to \infty$' (asymptotically separated ordinates). Some implications of the main results on the estimation of the index of dependence are also discussed.