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OPTIMAL BUFFER SIZE FOR A STOCHASTIC PROCESSING NETWORK WITH A DRIFT.

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ABSTRACT

We consider a one dimensional stochastic control problem that arises from queueing network applications. The state process corresponding to the queue-length is given by a stochastic differential equation which reflects at the origin. The controller can choose the drift coefficient which represents the service rate and the buffer size $b > 0$. When the queue-length reaches b, the new customers are rejected and this incurs a penalty. There are three types of costs involved: A control cost related to the dynamically controlled service rate, a congestion cost which depends on the queue-length and a rejection penalty for the rejection of the customers. We consider the problem of minimizing long term average cost (ergodic cost) criteria. We obtain an optimal drift rate (i.e an optimal service rate) as well as the optimal buffer size $b^* > 0$. When the buffer size $b > 0$ is fixed and where there is no congestion cost, this problem is similar to the work in [1]. Our method is quite different from that of [1]. To obtain a solution to the corresponding Hamilton-Jacobi-Bellman (HJB) equation, we analyze a family of ordinary differential equations and make use of some specific characteristics of this family of solutions to obtain the optimal buffer size $b^* > 0$.

[1] B. Ata and J. M. Harrison and L. A. Shepp. (2005). Drift rate control of a Brownian processing system. *The Annals of Applied Probability* 15 (2): 11451160.