

The Relationship between Confidence Intervals for Failure Probabilities and Life Time Quantiles

by

Yili Hong¹, William Q. Meeker¹, and Luis A. Escobar²
¹Iowa State University, ²Louisiana State University

September 04, 2007

ABSTRACT

The failure probability of a product $F(t)$ and the life time quantile t_p are commonly used metrics in reliability applications. Confidence intervals are used to quantify the statistical uncertainty of estimators of these two metrics. In practice, a set of pointwise confidence intervals for $F(t)$ or the quantiles t_p are often computed and plotted on one graph, which we refer to as pointwise “confidence bands.” These confidence bands for $F(t)$ or t_p can be obtained through normal approximation, likelihood, or other procedures. In this paper, we compare normal approximation and likelihood methods and introduce a new procedure to get the confidence intervals for $F(t)$ by inverting the pointwise confidence bands of the quantile t_p function. We show that it is valid to interpret the set of pointwise confidence intervals for the quantile function as a set of pointwise confidence intervals for $F(t)$ and vice-versa. Our results also indicate that the likelihood based pointwise confidence bands have desirable statistical properties, beyond those that were known previously.