

Chapter xx

Using Accelerated Tests to Predict Service Life in Highly-Variable Environments

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Abstract

Today's manufacturers need to develop newer, higher technology products in record time while improving productivity, reliability, and quality. This requires improved accelerated test (AT) methods that can usefully predict service life. For example, automobile manufacturers would like to develop a 3-month test to predict 5 or 10-year field reliability of a coating system. Such estimation/prediction from ATs involves *extrapolation*. Seriously inadequate predictions will result unless adequate models and methods are used. This paper describes a general framework within which one can use laboratory test results to predict product field service life performance of certain products in a highly-variable environment.

Introduction

Difficulty establishing correlation between laboratory tests and outdoor weathering tests for paints and coatings

Manufacturers of paints and coatings, for example, have had difficulty in establishing adequate correlation between their laboratory tests and field experience. Most of the laboratory tests attempt to accelerate time by "speeding up the clock." This is done by increasing average level of experimental factors like UV radiation, temperature, and humidity and cycling these experimental factors more rapidly than what is seen in actual use, in an attempt to simulate and accelerate outdoor aging. Such experiments violate some of the rules of good experimental design (e.g., by varying important factors together in a manner that confounds the effects of the factors) and the high levels of the accelerating variables can induce new failure modes. Thus such accelerated tests provide little fundamental understanding of the underlying degradation mechanisms. Because experience has shown that the results of these tests are unreliable, standard product evaluation for paints and coatings still requires outdoor testing in places like Florida and Arizona (hot humid and hot dry environments, respectively). Such testing, however, is costly and takes too much time.

Other possible reasons for differences between laboratory tests and outdoor weathering tests include

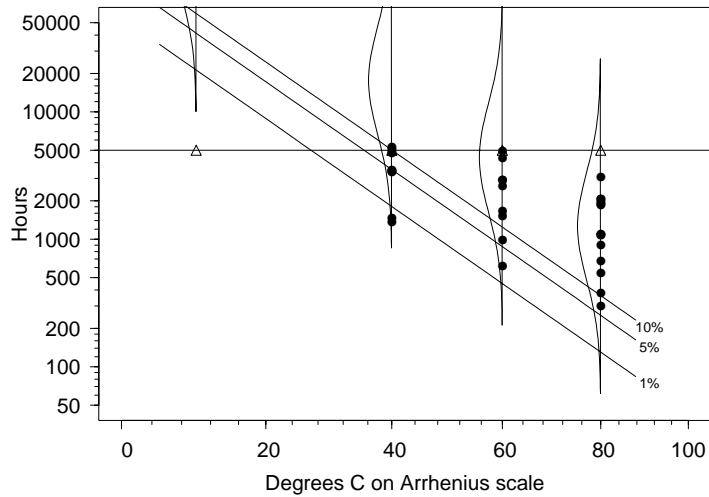


Figure 1: The Arrhenius-lognormal log-linear regression model fit to the Device-A ALT data.

- Inadequate control/monitoring of laboratory accelerated test conditions [e.g., $\xi = (\text{UV}, \text{temperature}, \text{humidity})$].
- Inadequate control/monitoring of field testing environmental conditions at outdoor exposure sites.
- Physical/chemical models that do not provide an adequate description of the relationship between degradation rates and experimental/environmental variables.
- Prediction models and methods that do not properly account for temporal environmental variability.

See [4] and [3] for a detailed description of issues relating to prediction of service life for paints and coatings.

Traditional Accelerated Tests

Reference [6] describes traditional accelerated life tests that are often used to provide timely information about life characteristics of components and materials. Other useful references for accelerated testing include [8] and Chapters 18 and 19 of [5]. This section briefly reviews these methods and provides an introduction to methods for using more powerful accelerated degradation tests.

An accelerated life test

Figure 1 shows the data from an accelerated life test on an electronic device (which we call Device A). These data were originally analyzed in [1]. The response was failure time for those devices that failed and the amount of running time for the others. The purpose of the experiment was to predict the early part of the failure-time distribution of the devices at an ambient use temperature of 10°C, presumably for a system to be installed under-sea. Superimposed on Figure 1 is a lognormal-

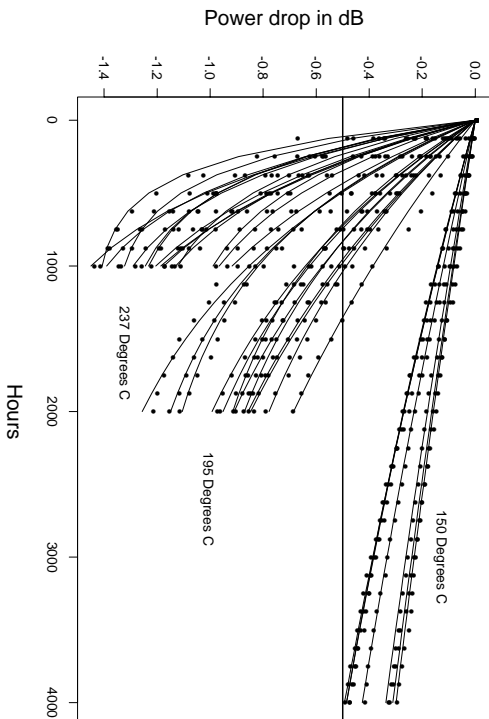


Figure 2: Device-B power drop accelerated degradation test data and fitted curves.

Arrhenius model that describes the failure time distribution for the devices as a function of ambient temperature.

The Arrhenius-lognormal regression model is

$$\Pr[T(\text{temp}) \leq t] = \Phi_{\text{nor}} \left[\frac{\log(t) - \mu}{\sigma} \right]$$

where Φ_{nor} is the standard normal cumulative distribution function, $\mu = \beta_0 + \beta_1 x$,

$$x = \frac{11605}{\text{temp K}} = \frac{11605}{\text{temp } ^\circ\text{C} + 273.15},$$

temp K is temperature Kelvin, and $\beta_1 = E_a$ is the effective activation energy.

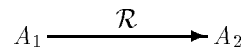
For this application, the use-environment conditions were stable and well characterized. See [1] and Chapter 19 of [5] for additional details on the model and the analyses of these data.

An accelerated degradation test

Degradation data, when they are available, provide more information than the more common failure-time data. Degradation data also offer advantages for developing mechanistic models that are important for accelerated testing and other applications requiring extrapolation outside the range of one's data. Chapters 13 and 21 of [5] describe methods for analyzing such data. We outline some of the basic concepts here.

Figure 2 shows the data from an accelerated degradation test on an RF power amplifier (which we call Device B). These data were originally analyzed in [7]. The Device B test is called a degradation test because the response was degradation in performance, in this case power drop, as a function of operating time and temperature. The temperatures shown are the junction temperatures corresponding to the environment in which the devices were operating. A device was defined to have failed after its power had dropped by 0.5 decibels (dB). The purpose of the degradation test was to estimate the fraction of units that would fail after 10 years at an operating junction temperature of 80°C.

The degradation in power was believed to have been caused by a mechanism that could be adequately described by single-step chemical reaction with rate constant \mathcal{R} . Diagrammatically,



The rate equations for this reaction are

$$\frac{dA_1}{dt} = -\mathcal{R}A_1 \quad \text{and} \quad \frac{dA_2}{dt} = \mathcal{R}A_1, \quad \mathcal{R} > 0 \quad (1)$$

and the power drop was believed to be proportional to the level of A_2 . Because power drop is observable, we use A_2 to denote this quantity. The solution of the system of differential equations in (1) gives

$$\begin{aligned} A_1(t) &= A_1(0) \exp(-\mathcal{R}t) \\ A_2(t) &= A_2(0) + A_1(0)[1 - \exp(-\mathcal{R}t)] \end{aligned} \quad (2)$$

where $A_1(0)$ and $A_2(0)$ are initial conditions. If $A_2(0) = 0$, letting $A_2(\infty) = \lim_{t \rightarrow \infty} A_2(t) = A_1(0)$, the solution for $A_2(t)$ is

$$A_2(t) = A_2(\infty)[1 - \exp(-\mathcal{R}t)]. \quad (3)$$

The asymptote $A_2(\infty)$ reflects the limited amount of a material that was available for reaction to the harmful compounds.

Reference [7] describes Device B degradation with the model in (3), assuming that random distributions on the asymptote $A_2(\infty)$ and the rate constant \mathcal{R} reflect unit-to-unit variability. The lines superimposed on Figure 2 are fitted regression curves, for each device, using the model in (3).

Simple temperature acceleration

The Arrhenius model describing the effect that temperature has on the rate of a simple first-order chemical reaction is

$$\mathcal{R}(\text{temp}) = \gamma_0 \exp \left[\frac{-E_a}{k_B \times (\text{temp} + 273.15)} \right] = \gamma_0 \exp \left(\frac{-E_a \times 11605}{\text{temp} + 273.15} \right) \quad (4)$$

where temp is temperature in $^{\circ}\text{C}$ and $k_B = 1/11605$ is Boltzmann's constant in units of electron volts per $^{\circ}\text{C}$. The pre-exponential factor γ_0 and the reaction activation energy E_a in units of electron volts are characteristics of the particular chemical reaction. Taking the ratio of the reaction rates at temperatures temp and temp_U cancels γ_0 giving an Acceleration Factor

$$\mathcal{AF}(\text{temp}, \text{temp}_U, E_a) = \frac{\mathcal{R}(\text{temp})}{\mathcal{R}(\text{temp}_U)} = \exp \left[E_a \left(\frac{11605}{\text{temp}_U + 273.15} - \frac{11605}{\text{temp} + 273.15} \right) \right] \quad (5)$$

that depends only on the two temperature levels and the activation energy. If $\text{temp} > \text{temp}_U$, then $\mathcal{AF}(\text{temp}, \text{temp}_U, E_a) > 1$. For simplicity, we use the notation $\mathcal{AF}(\text{temp}) = \mathcal{AF}(\text{temp}, \text{temp}_U, E_a)$ when temp_U and E_a are understood to be, respectively, product use (or other specified base-line) temperature and a reaction-specific activation energy.

The lines in Figure 3 were obtained by evaluating (3) at the estimates of the means of the distributions of the random asymptote and rate constant, as a function of temperature, using the Arrhenius model in (4).

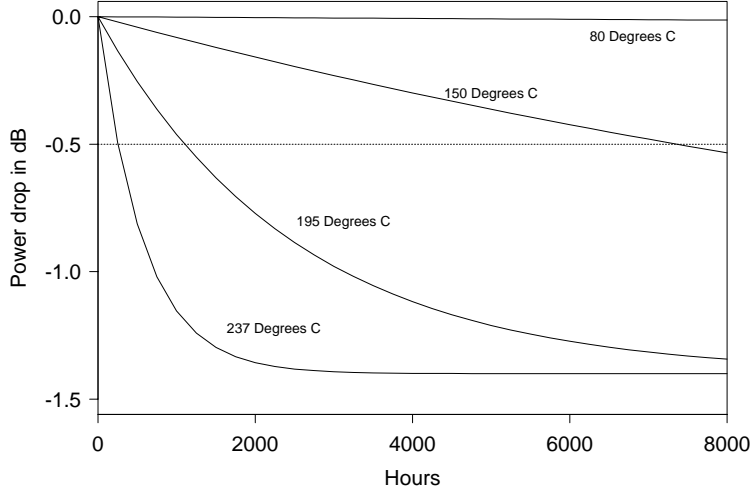


Figure 3: Device-B power drop predictions as a function of temperature.

General time transformation model

The Arrhenius model and other simple acceleration models result in what Meeker and Escobar [5] (Chapter 18) call a scale accelerated failure time (SAFT) model, in which the time to failure $T(\xi)$ at environment conditions ξ is related to the time to failure time $T(\xi_0)$ at environment conditions ξ_0 through the relationship

$$T(\xi) = T(\xi_0)/\mathcal{AF}(\xi).$$

Although this model is adequate for some simple situations, more complicated acceleration models are often needed (e.g., when a failure mechanism involves two important rate constants with different activation energies). A more general time-transformation function can be expressed as

$$T(\xi) = \Upsilon [T(\xi_0), \xi].$$

When the function $\Upsilon (t; \xi)$ is monotone increasing in t , the quantiles of the life distribution at ξ and ξ_0 are related by

$$t_p(\xi) = \Upsilon [t_p(\xi_0); \xi], \quad 0 < p < 1.$$

The cdfs at ξ and ξ_0 are related by

$$\begin{aligned} \Pr [T \leq t; \xi] &= \Pr [\Upsilon (T; \xi) \leq t; \xi_0] \\ &= \Pr [T \leq \Upsilon^{-1} (t; \xi); \xi_0]. \end{aligned}$$

A General Approach to Service Life Prediction in Complicated Environments

As described above, tests that simply “speed up the clock” have not provided adequate predictions of field performance of organic paints and coatings. In general, the mechanistic modeling of

failure and their dependency on accelerating variables is important for the successful application of accelerated testing. In order to do realistic service life prediction for complicated environments based on accelerated laboratory tests, we propose the following general approach, generalizing the traditional methods described in the previous section.

1. Use understanding of the physical/chemical mechanisms underlying product degradation and failure along with the experimental results to develop a deterministic product degradation model. To be workable, it will be necessary to develop a relatively simple model that, for example, identifies and focuses on the rate-limiting steps in the overall failure model.
2. Conduct laboratory experiments, using standard principles of experimental design, to gain fundamental understanding of the mechanisms leading to failure. Factors studied in the experiment should correspond to the environmental variables that affect service life.
3. Iterate between these first two steps in order to find and refine a model that will give the rate of degradation as a function of environmental variables and other important factors.
4. Use manufacturing process or experimental data to model and quantify product variability (e.g., unit-to-unit variability due to differences in raw materials and processing).
5. Use environmental time series data on the important factors that affect degradation (e.g., UV radiation, temperature, humidity) to characterize the environment. This could be done, for example, by identifying multivariate stochastic process models.
6. Use the variabilities in steps 4. and 5. along with the physical/chemical mechanism model identified in step 1. to define a stochastic process model for product degradation.
7. Use available data to estimate the unknown model parameters.
8. Use the product degradation model to generate a product service life prediction model.
9. Use statistical inference methods to quantify uncertainty in the service life distribution predicted from available data.

The remainder of this paper will develop and illustrate the basic modeling and prediction methods.

Degradation Model

Degradation, $\mathcal{D}(t)$, usually depends on environmental variables like **UV**, **temp**, and **RH**, that vary *over time*, say according to a multivariable profile $\boldsymbol{\xi}(t) = [\text{UV}, \text{temp}, \text{RH}, \dots]$. Laboratory tests are conducted in well-controlled environments (usually holding variables like UV, temperature, and humidity constant). Interest often centers, however, on life in a variable environment.

Modeling begins by developing a deterministic physical/chemical models for the failure mechanism. Then random and stochastic process distributions can be added, as needed, to account for important process variabilities (unit-to-unit, stochastic over time, or both). There are three situations to consider:

- The environmental conditions described by the vector $\boldsymbol{\xi}$ are constant over time.
- The environmental profiles $\boldsymbol{\xi} = \boldsymbol{\xi}(t)$ have a variable but *deterministic path* in time (i.e., an experimental step-stress vs. time profile).
- The environmental profiles $\boldsymbol{\xi} = \boldsymbol{\xi}(t)$ are random in time (e.g., outdoor/real-world weather conditions) and the distribution of environmental sample paths can be described by a (multivariate) *stochastic process* model with parameters $\boldsymbol{\theta}_{\boldsymbol{\xi}}$.

For example, Jorgensen et al. [2] used laboratory tests to identify a model similar to

$$\frac{d\mathcal{D}(t; L_{UV-B}, \mathbf{temp}, \text{RH})}{dt} = A \times L_{UV-B} \times \exp\left(-\frac{E_a}{k_B \times \mathbf{temp} \text{ K}}\right) \times \exp(C \times \text{RH})$$

where L_{UV-B} is the instantaneous UV irradiance in the UV-B band (290-320 nm), $\mathbf{temp} \text{ K}$ is temperature Kelvin, RH is relative humidity (all of the environmental variables are potentially functions of time), k_B is Boltzmann's constant, E_a is an activation energy, and A and C are other parameters characteristic of the material and the degradation process.

Deterministic degradation model

For a given environmental profile $\boldsymbol{\xi}(t)$, the cumulative degradation at time t for a particular unit (specified by a unit parameter vector $\boldsymbol{\beta}$) can be expressed deterministically as

$$\begin{aligned} \mathcal{D}(t) &= \int_0^t d\mathcal{D}[\tau, \boldsymbol{\xi}(\tau)] \\ &= \int_0^t \frac{d\mathcal{D}[\tau; \mathbf{UV}(\tau), \mathbf{temp}(\tau), \text{RH}(\tau), \dots]}{d\tau} d\tau \end{aligned} \quad (6)$$

where $d\mathcal{D}[\tau; \boldsymbol{\xi}(\tau)]/d\tau$ is the degradation rate and $\boldsymbol{\xi}(\tau)$ is the vector of environmental conditions at time τ (to simplify notation we suppress, for the moment, the dependence of $\mathcal{D}(t)$ on unit parameters $\boldsymbol{\beta}$). In general, the cumulative degradation paths $\mathcal{D}(t)$ differ from unit to unit due to:

- Intrinsic unit-to-unit differences (raw materials, processing differences).
- Extrinsic differences (e.g., in environmental profiles denoted by $\boldsymbol{\xi}(\tau)$).

Stochastic degradation model

The environmental variables in the profile $\boldsymbol{\xi}(t)$ are controlled in laboratory tests, but will be stochastic over time in actual product use. In order to evaluate the distribution of degradation paths for a stochastic environment, one can still use (6), but the integral becomes a stochastic integral.

A Simple Example

This section presents a simple example to show how to predict the effect of environmental variability on degradation. The example is based on the Device B power drop degradation model in (3). To keep the example simple, we will assume that there is no unit-to-unit variability (i.e., $\boldsymbol{\beta} = (\gamma_0 = 1.59 \times 10^{-13}, A_2(\infty) = -1.42)$ and $E_a = 0.72$ are held constant).

Figure 3 shows the predicted degradation paths for Device B power drop, according to the deterministic degradation model with Arrhenius temperature dependence, but with no unit-to-unit variability.

In order to predict power drop for a unit in which temperature (and thus degradation rate) changes over time, one can use the following generalization of the model in (2):

$$A_2(t) = A_2(0) + A_1(0) \left[1 - \exp\left(-\int_0^t \mathcal{R}[\mathbf{temp}(\tau)] d\tau\right) \right]. \quad (7)$$

where $\mathcal{R}[\mathbf{temp}(\tau)]$ is the degradation rate constant, as a function of temperature, and temperature is allowed to vary with time τ . For example, if the rate is \mathcal{R}_1 from 0 to t_1 and \mathcal{R}_2 thereafter, then

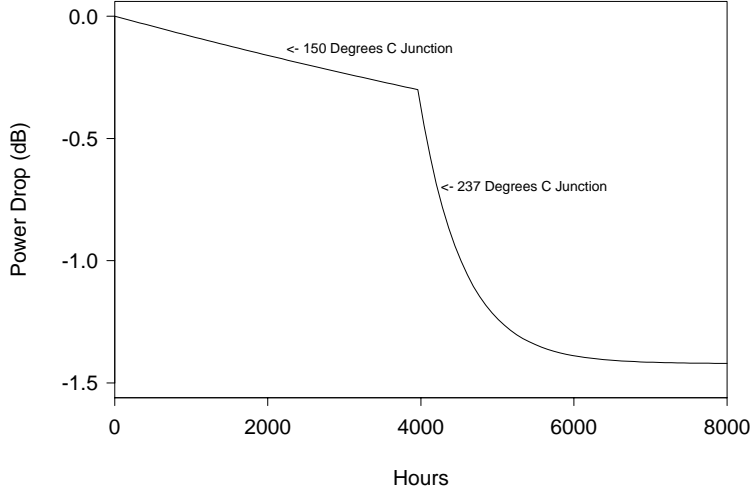


Figure 4: Power drop as a function of time with a change in temperature from 150°C to 237°C after 4000 hours.

the power drop profile would be

$$\begin{aligned} A_2(t) &= A_2(0) + A_1(0) [1 - \exp(-\mathcal{R}_1 t)], \quad 0 < t \leq t_1 \\ A_2(t) &= A_2(t_1) + A_1(t_1) [1 - \exp(-\mathcal{R}_2(t - t_1))], \quad t > t_1 \end{aligned}$$

where

$$A_1(t_1) = A_2(0) + A_1(0) - A_2(t_1).$$

Figure 4 shows the degradation path for a unit run at 150°C for 4000 hours and 237°C thereafter with $A_2(0) = 0$. For a general piece-wise constant temperature profile:

$$A_2(t) = A_2(t_{i-1}) + A_1(t_{i-1}) [1 - \exp(-\mathcal{R}_i(t - t_{i-1}))], \quad t_{i-1} < t \leq t_i, \quad (8)$$

where $i = 1, 2, \dots$, $\mathcal{R}_i = \mathcal{R}(\mathbf{temp}_i)$, \mathbf{temp}_i is the temperature between t_{i-1} and t_i , and $t_0 = 0$. Figure 5 shows the power drop path for a unit run at 150°C with a brief excursion to 237°C.

Figure 6 shows a simulated random temperature profile from Gaussian-noise discrete-time (one-hour time increments) first-order autoregressive [AR(1)] stochastic process model with a mean of 150°C, a standard deviation of 40°C, and autocorrelation $\rho_1 = 0.7$. Equation (8) provides a numerical tool for computing power drop for any arbitrary discrete-time temperature profile. The power drop profile corresponding to the temperature profile in Figure 6, computed from (8), is given in Figure 7.

Figure 8 shows simulated sample paths corresponding to five different simulated temperature profiles like that in Figure 6. The stochastic nature of simulated sample paths allows one to visualize the corresponding failure-time distribution. In Figure 8, for example, failure could be defined as the first time at which the power drop reaches -0.75 dB. Simulating a large number of such curves would provide an evaluation of the failure-time probability distribution.

The smooth curve in Figure 8 was obtained by substituting the average temperature (150°C) into the constant temperature power drop model (3). It is interesting to note that, in this example,

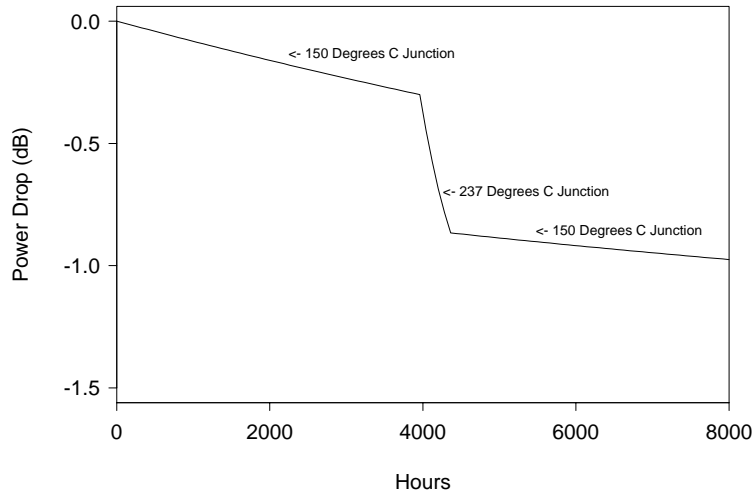


Figure 5: Power drop as a function of time with a brief excursion from 150°C to 237°C.

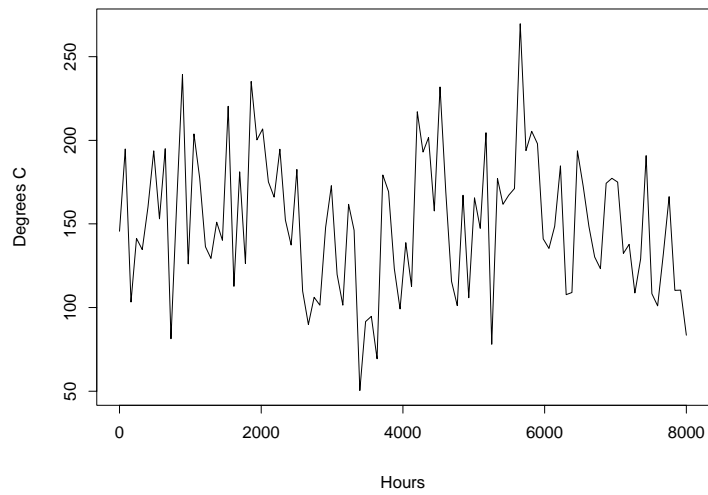


Figure 6: Simulated random temperature profile (AR(1) with $\rho_1 = 0.7$, $\mu = 150^\circ\text{C}$ and $\sigma = 40^\circ\text{C}$).

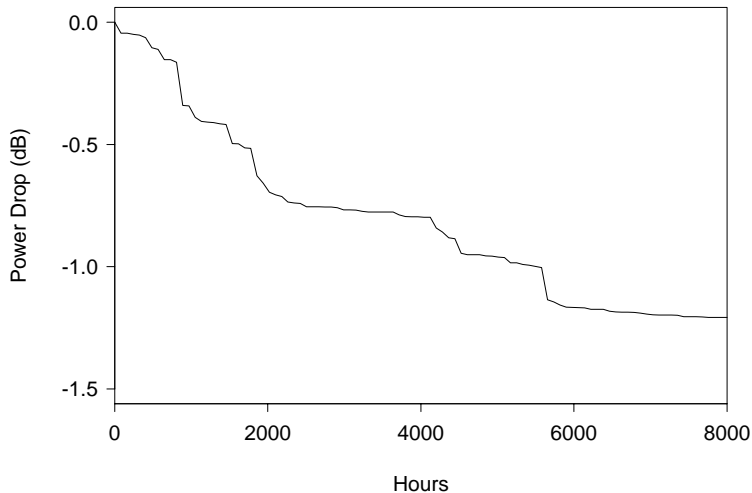


Figure 7: Power drop as a function of time corresponding to the simulated random temperature profile.

the stochastic sample paths tend to be lower than what would be predicted by substituting the average temperature into the constant temperature model. This shows the potential danger of substituting average values of random inputs into a deterministic model when trying to predict a response. The bias in the prediction is due to a combination of nonlinearity in (3) and in the Arrhenius relationship (4).

Service Life Distributions

In this section we return to the more general notation used at the beginning of this paper. As described in Chapter 13 of [5], a closed form equation for the failure-time distribution for a degradation model can be obtained only for some very special, simple degradation models, even when environmental conditions are constant over time. Numerical or simulation-based methods are generally used and several general approaches are described there.

This section extends the methods described in [5], providing expressions that apply with both unit-to-unit variability and random environmental profiles. We start by reviewing methods for deterministic environmental profiles.

Deterministic environmental profile

In this section we condition on a fixed environmental profile ξ and use $\mathcal{D}(t|\xi, \beta) = \mathcal{D}(t|\xi, \beta_1, \dots, \beta_k)$ to denote the degradation for a unit as a function of β and time t . For a specific unit, β contains parameters describing unit-to-unit variability. We suppose that in a population of units, β has the density $f(\beta)$. Note that for a given ξ and β , the path $\mathcal{D}(t|\xi, \beta)$ is deterministic. Conditional on the ξ , the environmental profile,

$$\begin{aligned}
 \Pr(T \leq t | \xi; \theta_\beta) &= F(t | \xi; \theta_\beta) = \Pr[\mathcal{D}(t | \xi; \beta) \leq \mathcal{D}_f; \theta_\beta] \\
 &= \int \delta[\mathcal{D}(t | \xi; \beta) \leq \mathcal{D}_f] f(\beta; \theta_\beta) d\beta
 \end{aligned} \tag{9}$$

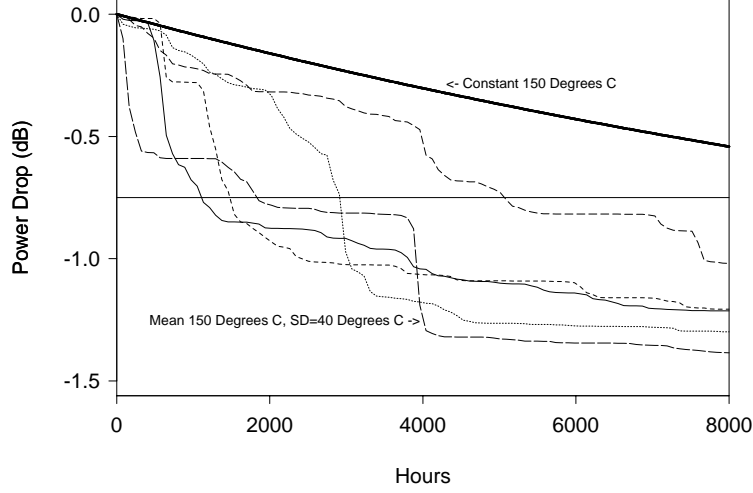


Figure 8: $\mathcal{D}(t|\xi)$ Power-drop cumulative degradation paths.

where the indicator function $\delta[\]$ is 1 if the argument is true and 0 otherwise. For a fixed environment profile $\xi = \xi_0$, the cdf $F(t|\xi_0; \theta_\beta)$ gives the fraction failing as a function of time, reflecting unit-to-unit variability.

As described in Section 13.5.2 of [5], when $\mathcal{D}(t; \beta)$ is a decreasing function of t , depending on just two parameters $\beta = (\beta_1, \beta_2)$ that are bivariate normal distributed, and if $\mathcal{D}(t; \beta)$ is decreasing in β_2 , then (notationally suppressing the dependency on the fixed ξ)

$$F(t; \theta_\beta) = P(T \leq t; \theta_\beta) = \int_{-\infty}^{\infty} \Phi \left[-\frac{g(\mathcal{D}_f, t, \beta_1) - \mu_{\beta_2|\beta_1}}{\sigma_{\beta_2|\beta_1}} \right] \frac{1}{\sigma_{\beta_1}} \phi \left(\frac{\beta_1 - \mu_{\beta_1}}{\sigma_{\beta_1}} \right) d\beta_1$$

where $g(\mathcal{D}_f, t, \beta_1)$ is the value of β_2 that gives $\mathcal{D}(t; \beta) = \mathcal{D}_f$ for specified β_1 and t and where

$$\begin{aligned} \mu_{\beta_2|\beta_1} &= \mu_{\beta_2} + \rho\sigma_{\beta_2} \left(\frac{\beta_1 - \mu_{\beta_1}}{\sigma_{\beta_1}} \right) \\ \sigma_{\beta_2|\beta_1}^2 &= \sigma_{\beta_2}^2 (1 - \rho^2). \end{aligned}$$

This approach is easy to adapt to functions $\mathcal{D}(t; \beta)$ that increase in β_2 or t . Also, this approach can, in principle, be extended in a straightforward manner when there are more than two continuous random variables. The amount of computational time needed to evaluate the multidimensional integral will, however, increase exponentially with the dimension of the integral.

Stochastic environmental profile

In this section suppose that the variability in the environmental profile $\xi(t)$ can be described by a stochastic process model (say with controlling parameters θ_ξ). Then, because the degradation rate depends on the environment, $\mathcal{D}(t; \beta)$ is also stochastic, as illustrated in Figure 8 with the power drop example. Considering the variability in the environmental profile ξ , $F(t|\xi; \theta_\beta)$ is a random function. For fixed time t_0 , $W = F(t_0|\xi; \theta_\beta)$ is a random variable reflecting the variability in the random environmental profiles.

Under the assumption that the random environment in ξ is independent of the unit-to-unit variability in β , the probability distribution

$$F_W(w; \theta_\beta, \theta_\xi) = \Pr(W \leq w) = \Pr[F(t_0 | \xi; \theta_\beta) \leq w], \quad 0 < w < 1$$

allows assessment of the distribution of fraction failing by t_0 , relative to an uncertain future environment. With respect to variability in the environment mental profile ξ , the expectation of W is

$$E(W) = E[F(t_0 | \xi; \theta_\beta)] = F(t_0; \theta_\beta, \theta_\xi)$$

giving the conditional probability of failure before time t_0 for a single unit. More generally,

$$\begin{aligned} F(t; \theta_\beta, \theta_\xi) &= \int \Pr[\mathcal{D}(t) \leq \mathcal{D}_f | \xi; \theta_\beta] f(\xi; \theta_\xi) d\xi \\ &= \int F(t | \xi; \theta_\beta) f(\xi; \theta_\xi) d\xi \end{aligned}$$

which is, in effect, averaging over all possible environmental profiles in the environment described by the environmental model parameters θ_ξ .

An alternative representation for the unconditional probability of failure

Conditional on a fixed β , the failure-time distribution is

$$F(t | \beta; \theta_\xi) = \Pr(T \leq t | \beta; \theta_\xi) = \Pr[\mathcal{D}(t) \leq \mathcal{D}_f] = \Pr\left[\int_0^t \frac{d\mathcal{D}[\tau; \xi(\tau)]}{d\tau} d\tau \leq \mathcal{D}_f\right]. \quad (10)$$

As in (6), computation of $F(t | \beta; \theta_\xi)$ requires the solution of the stochastic integral for the specific environment characterized by θ_ξ . Accounting for unit-to-unit variability in the β parameters gives the following alternative representation for the unconditional probability of failure for a single unit.

$$F(t; \theta_\beta, \theta_\xi) = \int F(t | \beta; \theta_\xi) f(\beta; \theta_\beta) d\beta. \quad (11)$$

Time transformation function with stochastic environmental profiles

As shown in the previous section, for a model with stochastic process variability, the failure-time distribution can be determined by solving stochastic differential equations or averaging of the possible realizations of a random environment. In principle, it is also possible to determine a time transformation function Υ for any specified failure model, characterized by a degradation model and a failure criterion, \mathcal{D}_f and such a transformation function can be generalized to allow for stochastic environments. Such a time transformation function can be used to relate the failure-time distributions at two different locations with different environments characterized by different set of stochastic process model parameters, say θ_ξ and θ_{ξ_0} . That is,

$$\begin{aligned} \Pr[T \leq t; \theta_\xi, \theta_\beta] &= \Pr[\Upsilon(T; \theta_\xi) \leq t; \theta_{\xi_0}, \theta_\beta] \\ &= \Pr[T \leq \Upsilon^{-1}(t; \theta_\xi); \theta_{\xi_0}, \theta_\beta]. \end{aligned}$$

The function $\Upsilon(t; \theta_\xi)$ can be determined by finding the mapping between the failure time quantiles for different $0 < p < 1$ at conditions characterized by a stochastic process with parameters θ_ξ versus those with parameters θ_{ξ_0} .

Concluding Remarks and Areas for Future Research

This paper has outlined a general framework for using accelerated degradation tests to predict the performance in a highly-variable environment. The methods depend importantly on the ability to describe adequately the failure process by a relatively simple model (so that its parameters can be estimated reliably from experimental data) that gives degradation rate as a function of environmental conditions. Such models will have to be provided by scientists working with particular materials and products. Even with such a model in hand, there remain a number of technical challenges that are the subject of current research. These include

- Modeling of environmental variables (e.g., UV and other weather-related variables) with a stochastic process model.
- Methods for quantifying uncertainty in forecasts due to:
 - ▶ Uncertain future weather.
 - ▶ Uncertainty in model parameters (for both the random-effect unit-to-unit variability and the weather models) due to limited data.

Prediction intervals for quantities of interest like cumulative degradation or fraction failing would be the natural means of describing the effects of this uncertainty.

- Numerical techniques for implementing the methods.
- Approximations that will allow rapid analyses, at least for some special-case situations.

Acknowledgments

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