ACCELERATED DESTRUCTIVE DEGRADATION TESTS: DATA, MODELS, AND ANALYSIS

Luis A. Escobar
Dept. of Experimental Statistics
Louisiana State University
Baton Rouge, LA 70803, USA

William Q. Meeker
Dept. of Statistics
Iowa State University
Ames, IA 50011, USA

Danny L. Kugler and Laura L. Kramer
Imaging & Printing Group
Hewlett-Packard
Corvallis, OR 97330, USA

Abstract

Degradation data analysis is a powerful tool for reliability assessment. Useful reliability information is available from degradation data when there are few or even no failures. For some applications the degradation measurement process destroys or changes the physical/mechanical characteristics of test units. In such applications, only one meaningful measurement can be can be taken on each test unit. This is known as "destructive degradation." Degradation tests are often accelerated by testing at higher than usual levels of accelerating variables like temperature.

This chapter describes an important class of models for accelerated destructive degradation data. We use likelihood-based methods for inference on both the degradation and the induced failure-time distributions. The methods are illustrated with the results of an accelerated destructive degradation test for an adhesive bond.

1 Introduction

1.1 Motivation

Today's manufacturers face strong pressure to develop newer, higher technology products in record time. In addition, there are competitive pressures to improve productivity, product field reliability, and overall quality. This implies the increased need for up-front testing of materials, components and systems. Traditional life tests (where time to failure is the response) may result in few or

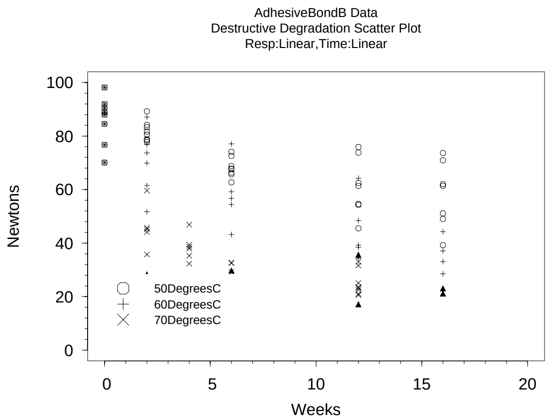

Figure 1: Adhesive Bond B ADDT data scatter plot.

no failures, even when accelerated (e.g., by testing at higher-than-usual levels of temperature or voltage). Accelerated degradation tests can be useful for such up-front testing.

1.2 Advantages and Difficulties of Degradation Data

Degradation is the natural response for some tests. With degradation data, it is possible to make useful reliability inferences, even with no failures. In addition, modeling degradation data provides more justification and credibility for extrapolative acceleration models. This is because modeling is closer to the physics-of-failure mechanisms.

It may, however, be difficult, costly, or impossible to obtain degradation measures from some components or materials. Often taking degradation measures will require destructive measurements, which is the motivation for the current work. Also, the analysis of destructive degradation data generally requires the use of special software. For more information on degradation data (with special emphasis on repeated measures degradation), see Chapters 13 and 21 of Meeker and Escobar[3].

1.3 Adhesive Bond B ADDT Data

The objective of the experiment was to assess the strength of an adhesive bond over time. In particular, there is interest in estimating the proportion of devices with a strength below 40 Newtons after 5 years of operation (approximately 260 weeks) at room temperature of 25°C. The test needed to be completed in 16 weeks, and thus acceleration would be needed, as little or no degradation could be expected at 25°C during the length of the test. The test is destructive; strength can be measured only once on each unit.

Table 1: Adhesive Bond B test plan.

Temp	Weeks Aged					
°C	0	2	4	6	12	16
70		6	6	4	9	0
60		6	0	6	6	6
50		8	0	8	8	7
—	8					

The strength data are shown in Figure 1 (the data have been modified by a change in scale, in effect changing the units of strength, in order to protect proprietary information). There were six strength measurements at three different levels of temperature that had suspiciously low values. Fitting a model to accommodate these values gave estimates that the engineers involved in the problem knew to be inconsistent with actual failure probabilities. It was believed all of these lower readings could be attributed to the fabrication of the test units and that this problem could be avoided in actual production (a test would have to be conducted to verify that this was so). Thus, for purposes of modeling and analysis, it was decided to mark these observations as right-censored values, indicating that the actual level of strength is unknown, but certainly larger than the recorded value. In our plots, such observations are marked with a ▲.

The experiment included 8 units that were measured at the start of the experiment, with no aging. A total of 80 additional units were aged and measured according to the temperature and time schedule shown in Table 1.

1.4 Related Literature

Chapter 7 of Tobias and Trindade[12], Chapters 13 and 21 of Meeker and Escobar[3], and Meeker, Escobar, and Lu[5] present statistical methods for estimating a failure-time distribution from repeated measures degradation. The important pioneering work of Nelson[8] and Chapter 11 of Nelson[9] describe methods for using destructive degradation data to estimate performance degradation and related failure time distributions for an insulation, using a model that is a special case of the model that is used in this chapter. This chapter provides some useful generalizations, further technical details, a new application for destructive degradation data. In particular, we show how to deal with censored data, multiple accelerating variables, describe model identification and diagnostic tools, provide more details on the distribution of failure times, and provide discussion of acceleration factors.

1.5 Overview

This remainder of this chapter is organized as follows. Section 2 describes the degradation model that we use for destructive degradation. Section 3 outlines methods for ML estimation with right-censored data, both for individual test conditions and the full acceleration model. Section 4 gives formulas for the distribution of degradation. Section 5 gives formulas for the failure time distribution induced by the degradation model. Section 6 shows how to compute and interpret acceleration factors. Section 7 contains some concluding remarks and areas for future research.

2 Model

2.1 Model for a Degradation Path

The model for the actual degradation path of a unit at time t_i and a particular accelerating variable condition AccVar_j (e.g., temperature) is

$$\mathcal{D}_{ij} = \mathcal{D}(\tau_i, x_j, \boldsymbol{\beta})$$

where $\tau_i = h_t(t_i)$ and $x_j = h_a(\text{AccVar}_j)$ are known monotone increasing transformations of t_i and AccVar_j, respectively. The form of the function $\mathcal{D}$ and the appropriate transformations may be suggested by physical-chemical theory, (see, for example Meeker and LuValle[6] and Meeker, Escobar, and Lu[5], past experience, or the data. When there is no possibility of confusion, τ_i and x_j are called the time and the AccVar condition, respectively. Rates in the model are with respect to transformed time $\tau = h_t(t)$. In our application, the path parameters $\boldsymbol{\beta}$ are fixed but unknown.

2.2 Model for Degradation Sample Paths

For unit k at time τ_i and accelerating variable condition x_j the sample path model is

$$y_{ijk} = h_d(\mathcal{D}_{ij}) + \epsilon_{ijk} = \mu_{ij} + \epsilon_{ijk}$$

where $\mu_{ij} = h_d(\mathcal{D}_{ij})$ and y_{ijk} are, respectively, monotone increasing transformations of $\mathcal{D}_{ij}$ and the observed degradation. ϵ_{ijk} is a residual deviation which describes unit-to-unit variability with $(\epsilon_{ijk}/\sigma) \sim \Phi(z)$, where $\Phi(z)$ is a completely specified distribution, for example $\Phi(z) = \Phi_{\text{nor}}(z)$ provides a normal model and $\Phi(z) = \Phi_{\text{sev}}(z)$ provides a smallest extreme value model.

Figure 2 suggests that some kind of transformation should be used to linearize the degradation paths. Several transformations were investigated on the Adhesive Bond B data and other similar data sets. In all cases the combination of a log transformation on degradation response and square root on time provided the best fit. Figure 3 shows the effect of using these transformations. Later we learned that this combination of transformations can be explained by the fact that the rate of the failure-causing degradation mechanism is controlled

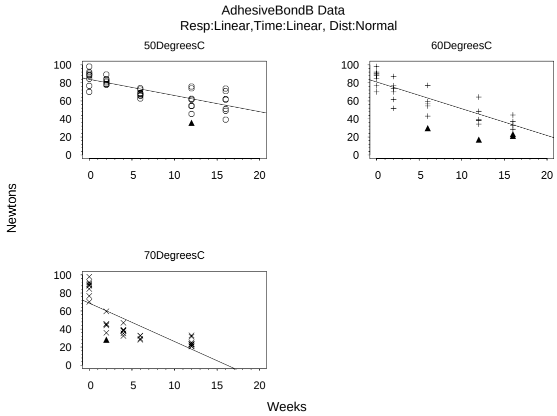

Figure 2: Adhesive Bond B ADDT data scatter plot at individual levels of temperature.

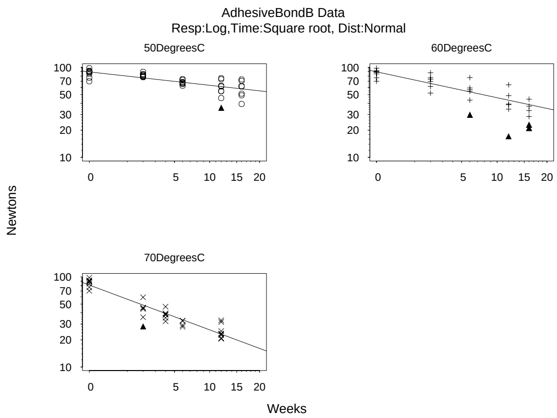

Figure 3: Individual normal distribution ML fits for the Adhesive Bond B ADDT data.

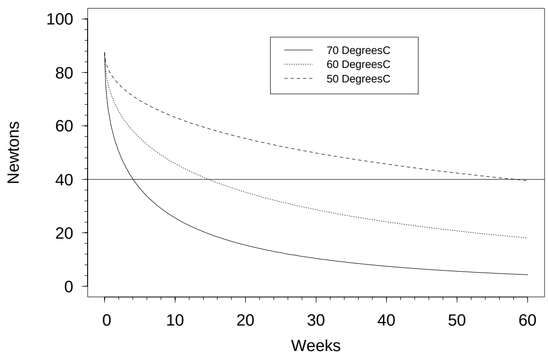

Figure 4: Degradation path model for three different temperatures.

by a process that can be described by the 2nd Law of Diffusion (otherwise known as Fick's Law).

2.3 Model for Acceleration

For the Adhesive Bond B application, the degradation rate (on the transformed time scale) is assumed to be described by the Arrhenius relationship. That is,

$$\mathcal{D}(\tau, x, \boldsymbol{\beta}) = \exp[\beta_0 + \beta_1 \exp(\beta_2 x)\tau]$$

where $\tau = \sqrt{\text{Weeks}}$ and $x = -11605/(^{\circ}\text{C}_j + 273.15)$ is Arrhenius-transformed temperature. This relationship between degradation and temperature is depicted in Figure 4. On the log-degradation scale, the individual regressions are linear in square root time τ. In particular, $\log[\mathcal{D}(\tau, x, \boldsymbol{\beta})] = \beta_0 + \beta_1 \exp(\beta_2 x)\tau$. These degradation sample-path models are, in general, of the form

$$\begin{aligned} y_{ijk} &= \mu_{ij} + \epsilon_{ijk} \\ &= \beta_0 + \beta_1 \exp(\beta_2 x_j)\tau_i + \epsilon_{ijk} \end{aligned} \tag{1}$$

where y_{ijk}, τ_i, and x_j may be monotone transformations of the measured degradation, t_i, and the AccVar variable, respectively. This degradation model is linear in the sense that for a specified AccVar condition x_j, the degradation is linear in τ_i. For multiple AccVar situations (e.g., temperature and humidity), the term $\beta_2 x_j$ can be replaced by a linear combination of accelerating variables $\boldsymbol{\beta}_2' \boldsymbol{x}_j$. In either case, however, the regression model with the acceleration terms is nonlinear in the unknown parameter(s) $\boldsymbol{\beta}_2$. Thus, even if there is no censoring, ordinary least squares cannot be used to estimate, simultaneously, all of the parameters of the model.

2.4 Interpretation of the Parameters

The interpretation of the degradation model parameters can be described as follows. For the linear degradation model in (1), β_0 is the degradation level when $\tau_i = 0$. For example, if $\tau_i = \sqrt{t_i}$ (or some other power transformation of time), then β_0 is degradation at time $t = 0$. If $\tau_i = \log(t_i)$ then β_0 is degradation at time $t = 1$. The degradation rate at AccVar level x_j is $\upsilon(x_j) = \beta_1 \exp(\beta_2 x_j)$. The sign ($\pm$) of β_1 determines whether degradation is increasing or decreasing in time. This rate is with respect to transformed degradation y and transformed time τ. For a power transformation of time $\tau = t^\kappa$ the parameter β_2 is related to the amount of acceleration obtained by increasing the accelerating variable AccVar.

2.5 Another ADDT Model Example

Chapter 11 of Nelson describes an example involving a destructive degradation experiment to assess insulation breakdown voltage as a function of temperature and time. The model is the same as that given in (1) with

$$\begin{aligned} y_{ijk} &= \log[(\text{Breakdown Voltage})_{ijk}] \\ \tau_i &= t_i = \text{Weeks}_i, \quad x_j = -11605/(^\circ\text{C}_j + 273.15) \\ (\epsilon_{ijk}/\sigma) &\sim \Phi_{\text{nor}}(z) \end{aligned}$$

and $\Phi_{\text{nor}}(z)$ is a standardized normal cdf.

3 Maximum Likelihood Estimation

3.1 Estimation at Individual Conditions

For the data at a fixed condition x_j of the AccVar with exact failure times and right-censored observations, the likelihood is

$$L_j(\boldsymbol{\theta}|\text{DATA}) = \prod_i \prod_{k=1}^{n_{ij}} \left[\frac{1}{\sigma}\phi\left(\frac{y_{ijk} - \mu_{ij}}{\sigma}\right)\right]^{\delta_{ijk}} \times \left[1 - \Phi\left(\frac{y_{ijk} - \mu_{ij}}{\sigma}\right)\right]^{1-\delta_{ijk}} \qquad (2)$$

where $\mu_{ij} = \mu(\tau_i, x_j, \boldsymbol{\beta}) = \beta_0 + \beta_1 \exp(\beta_2 x_j)\tau_i$, δ_{ijk} indicates whether observation y_{ijk} is a failure ($\delta_{ijk} = 1$) or a right censored observation ($\delta_{ijk} = 0$), $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \sigma)$ is the vector of unknown parameters, and n_{ij} is the number of observations at $(\tau_i,\ x_j)$. The logarithm of (2) can be maximized by using standard numerical function maximization methods. For fixed x_j, the identifiable parameters are the standard deviation of the error term, σ, the intercept β_0, and the slope of the line $\upsilon^{[j]} = \beta_1 \exp(\beta_2 x_j)$. Then for each specified condition of the AccVar x_j, three individual ML estimates are obtained, say $\widehat{\beta}_0^{[j]}$, $\widehat{\upsilon}^{[j]}$, and $\widehat{\sigma}^{[j]}$. The parameter $\upsilon^{[j]}$ can be interpreted as the degradation rate of μ_{ij} with respect to transformed time τ_i.

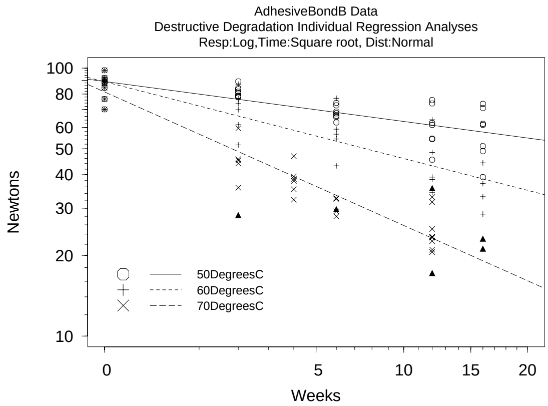

Figure 5: Overlay of individual normal distribution fits for the Adhesive Bond B ADDT data.

Table 2: Normal distribution individual parameter ML estimates and 95% confidence intervals for the slope at each level of temperature.

	ML Estimates			95% Approximate Confidence Interval for $\upsilon^{[j]}$	
AccVar_j	$\widehat{\beta}_0^{[j]}$	$\widehat{\upsilon}^{[j]}$	$\widehat{\text{se}}_{\widehat{\upsilon}^{[j]}}$	Lower	Upper
50°C	4.490	−0.1088	0.01494	−0.1424	−0.08309
60°C	4.489	−0.2089	0.02214	−0.2571	−0.16969
70°C	4.400	−0.3626	0.01944	−0.4028	−0.32643

Figure 3 shows the data and the individual ML regression line estimates for the three different temperature levels in the Adhesive Bond B example using model (1) with a normally distributed residual component, i.e., $\Phi(z) = \Phi_{\text{nor}}(z)$, y_{ijk} in the log-degradation scale, and the time τ_i in the square root scale. Figure 5 shows the same results, all on one plot. The parameter estimates for the simple regression model fit to the censored Adhesive Bond B data are given in Table 2. The standard errors were obtained by using local information (see, for example, Appendix Section B.6.4 in Meeker and Escobar[3]).

3.2 Arrhenius Plot of Degradation Rates

The ML estimates $\widehat{\upsilon}^{[j]}$ (slopes of the individual lines) can be used to identify the relationship between degradation rate and the AccVar. When the degradation is decreasing, use absolute values of the degradation rate. Because $\log(|\ \upsilon^{[j]}\ |) = \log(|\ \beta_1\ |) + \beta_2 x_j$ a plot of $\log(|\ \widehat{\upsilon}^{[j]}\ |)$ versus x_j should be approximately linear if the model relating the degradation rate and the AccVar level x_j is adequate.

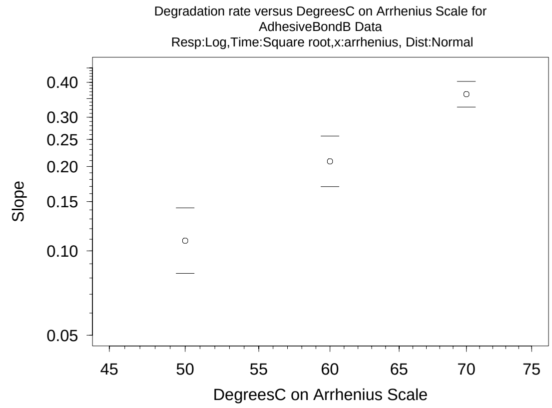

Figure 6: Arrhenius plot of individual degradation rates normal distribution ML estimates.

Figure 6 plots the absolute values of the ML estimates of the regression-line slopes at each individual level of temperature versus temperature on log-Arrhenius paper (equivalent to plotting the logarithm of the slopes versus $-11605/(°\mathrm{C}+273.15)$. The approximate 95% pointwise confidence intervals for the slopes aid in the interpretation of the plot, relative to the statistical importance of deviations from linearity. The nearly linear relationship in Figure 6 suggests good agreement with the Arrhenius model for temperature acceleration, at least within the range of the data.

3.3 Likelihood for the Acceleration Model Using All Data

For a sample of n units consisting of exact failure times and right-censored observations, the likelihood can be expressed as

$$\begin{aligned} L(\boldsymbol{\theta}|\mathrm{DATA}) &= \prod_{j} L_j(\boldsymbol{\theta}|\mathrm{DATA}) \qquad (3) \\ &= \prod_{ijk} \left[\frac{1}{\sigma}\,\phi\left(\frac{y_{ijk}-\mu_{ij}}{\sigma}\right)\right]^{\delta_{ijk}} \times \left[1-\Phi\left(\frac{y_{ijk}-\mu_{ij}}{\sigma}\right)\right]^{1-\delta_{ijk}} \end{aligned}$$

where $\boldsymbol{\theta} = (\beta_0, \beta_1, \beta_2, \sigma)$, $\mu_{ij} = \beta_0 + \beta_1 \exp(\beta_2 x_j)\tau_i$, $x_j = -11605/(°\mathrm{C}_j + 273.15)$, and δ_{ijk} indicates whether observation ijk is a failure ($\delta_{ijk} = 1$) or a right censored observation ($\delta_{ijk} = 0$).

For the Adhesive Bond B data, Figure 7 shows the ML estimates from the combined data for each of the three levels of temperature used in the experiment plus the use condition of 25°C. Note that all of the lines cross at the common intercept at time 0. The parameter estimates for the acceleration model fit to the Adhesive Bond B data are given in Table 3. Again, standard errors are based on local information.

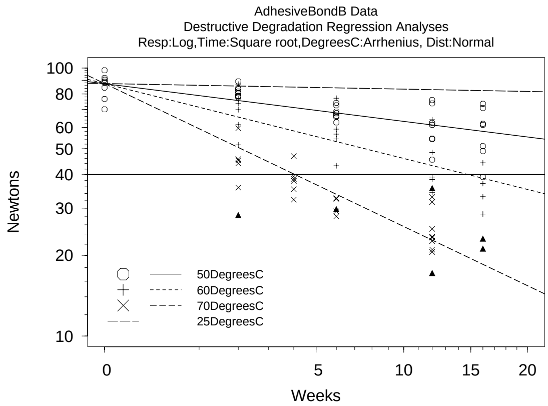

Figure 7: Normal distribution Arrhenius model fit to the Adhesive Bond B ADDT data

Table 3: ML estimates for the acceleration model fit to the Adhesive Bond B data.

Parameter	ML Estimate	Standard Error	95% Approximate Confidence Interval Lower	Upper
β_0	4.471	0.03864	4.396	4.547
β_1	-8.641×10^8	1.595×10^9	-3.989×10^9	2.261×10^9
β_2	0.6364	0.05488	0.5375	0.7536
σ	0.1580	0.01233	0.1356	0.1841

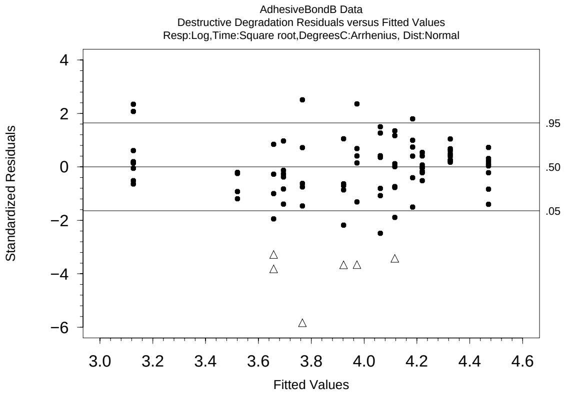

Figure 8: Adhesive Bond B ADDT data residuals versus fitted values

3.4 Residual Analysis

Analysis of residuals to detect model departures is just as important for the destructive degradation models as it is for other regression models. The censored observations can make interpretation of such plots more complicated. We follow the approach in Nelson[7], yielding a right-censored residual corresponding to each right-censored degradation reading.

For the Adhesive Bond B data, Figure 8 shows a plot of the residuals versus fitted values (there is one distinct fitted value for each time/temperature combination). Here the $\triangle$ symbol indicates the position of the right-censored residuals. The actual (unobserved) residuals would be larger than these values. Figure 9 is a normal probability plot of the residuals (based on an adjusted Kaplan-Meier estimate computed from the right-censored residuals). This plot suggests that the normal distribution provides a good description of the residuals.

4 Distribution of Degradation at Time and AccVar Conditions (t, AccVar)

4.1 Degradation Distribution CDF

For given time and AccVar conditions (t, AccVar), the degradation distribution is

$$F_Y(y;\tau,x) = P(Y \le y;\tau,x) = \Phi\left[\frac{y-\mu(\tau,x,\boldsymbol{\beta})}{\sigma}\right]$$

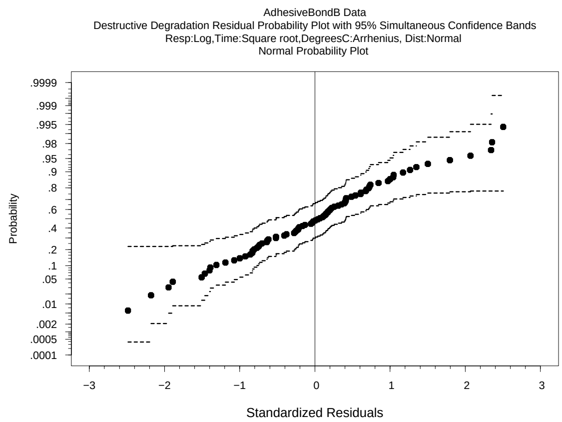

Figure 9: Adhesive Bond B ADDT data normal distribution residual probability plot

where $y = h_d(\text{degradation})$, $\mu(\tau, x, \boldsymbol{\beta}) = \beta_0 + \beta_1 \exp(\beta_2 x)\tau$. The ML estimate of the degradation distribution for given (t, AccVar) is

$$\widehat{F}_Y(y; \tau, x) = \Phi\left(\frac{y - \widehat{\mu}}{\widehat{\sigma}}\right)$$

where $\widehat{\mu} = \widehat{\beta}_0 + \widehat{\beta}_1 \exp(\widehat{\beta}_2 x)\tau$, $\tau = h_t(t)$, $x = h_a(\text{AccVar})$, and $\widehat{\beta}$'s are ML estimates.

For the Adhesive Bond B data, the normal distribution ML estimate of $F_Y(y; \tau, x)$ at time and temperature (Weeks, °C) is

$$\widehat{F}_Y(y; \tau, x) = \Phi_{\text{nor}}\left(\frac{y - \widehat{\mu}}{\widehat{\sigma}}\right)$$

where $\widehat{\mu} = \widehat{\beta}_0 + \widehat{\beta}_1 \exp(\widehat{\beta}_2 x)\tau$, $\tau = \sqrt{\text{Weeks}}$, $x = -11605/(°\text{C} + 273.15)$. The ML estimates $\widehat{\beta}_0$, $\widehat{\beta}_1$, $\widehat{\beta}_2$, and $\widehat{\sigma}$ are given in Table 3.

4.2 Degradation Distribution Quantiles

The p quantile of the degradation distribution is $y_p = \mu(t, x, \boldsymbol{\beta}) + \sigma\Phi^{-1}(p)$. The ML estimate of the p quantile on the transformed scale (log Newtons for the Adhesive Bond B example) is

$$\widehat{y}_p = \widehat{\mu} + \widehat{\sigma}\Phi^{-1}_{\text{nor}}(p).$$

5 Induced Failure Time Distribution at Fixed Values of $(\textbf{AccVar}, \mathcal{D}_{\text{f}})$ for Decreasing Linear Degradation

5.1 Failure Time CDF

Observe that $T \leq t$ [i.e., $h_t(T) \leq \tau$] is equivalent to observed degradation being less than $\mathcal{D}_{\text{f}}$ (i.e., $Y \leq \mu_{\text{f}}$,) where $\mu_{\text{f}} = h_d(\mathcal{D}_{\text{f}})$. Then

$$
\begin{aligned}
F_T(t; x, \boldsymbol{\beta}) &= \Pr(T \leq t) \\
&= F_Y(\mu_{\text{f}}; x, \boldsymbol{\beta}) = \Phi\left[\frac{\mu_{\text{f}} - \mu(\tau, x, \boldsymbol{\beta})}{\sigma}\right] \\
&= \Phi\left(\frac{\tau - \nu}{\varsigma}\right), \text{ for } t \geq 0
\end{aligned} \tag{4}
$$

where $\tau = h_t(t)$,

$$\nu = \frac{(\beta_0 - \mu_{\text{f}}) \exp(-\beta_2 x)}{\mid \beta_1 \mid} \quad \text{and} \quad \varsigma = \frac{\sigma \exp(-\beta_2 x)}{\mid \beta_1 \mid}.$$

The failure time distribution in (4) is a mixed distribution with a *spike* of probability, $\Pr(T = 0) = \Phi\left[(\beta_0 - \mu_{\text{f}})/\sigma\right]$ at $t = 0$. For $t > 0$ the cdf is continuous and it agrees with the cdf of a log-location-scale variable with standardized cdf $\Phi(z)$, with location parameter ν and scale parameter ς.

For the Adhesive Bond B application, $\mathcal{D}_{\text{f}} = 40$. Figure 10 provides a visualization of the failure-time distribution induced by the degradation model at 25°C, based on the ML estimates of the Adhesive Bond B example. The figure shows clearly the reason for the spike of probability at time zero.

5.2 Failure Time Distribution Quantiles

The p quantile of the failure time distribution can be expressed as follows. Let $p \geq \Phi\left[(\beta_0 - \mu_{\text{f}})/\sigma\right]$ and

$$h_t(t_p) = \tau_p = \nu + \varsigma\Phi^{-1}(p) \tag{5}$$

where

$$\nu = \frac{(\beta_0 - \mu_{\text{f}}) \exp(-\beta_2 x)}{\mid \beta_1 \mid} \quad \text{and} \quad \varsigma = \frac{\sigma \exp(-\beta_2 x)}{\mid \beta_1 \mid}.$$

Then the p quantile of the failure time distribution is $t_p = h_t^{-1}\left[\nu + \varsigma\Phi^{-1}(p)\right]$. Substituting the expressions for ν and ς into (5), taking the logarithm, and simplifying gives

$$\log[h_t(t_p)] \quad = \quad \log(\tau_p) = -\beta_2 x + \log\left[\frac{(\beta_0 - \mu_{\text{f}}) + \sigma\Phi^{-1}(p)}{\mid \beta_1 \mid}\right].$$

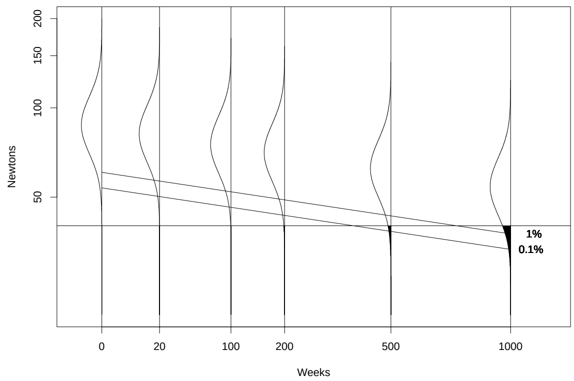

Figure 10: ML estimate showing proportion failing as a function of time at condition 25°C

This shows that the log of the transformed failure-time distribution quantiles are linear in the transformed AccVar condition x. If $p < \Phi\left[(\beta_0 - \mu_{\mathrm{f}})/\sigma\right]$, the p quantile of the failure time distribution is 0.

For the Adhesive Bond B example, Figure 11 is a model plot showing ML estimates of the failure-time distribution quantiles as a function of temperature on log-Arrhenius scales.

6 Acceleration Factors

Those who conduct accelerated tests often need to quote an "acceleration factor" to indicate the amount of time being saved by acceleration. Here we consider acceleration factors for time power transformations (i.e., $\tau = h_t(t) = t^{\kappa}$, where $\kappa > 0$). To obtain the effect of acceleration due to using higher than usual values of the AccVar x, let $\tau(x)$ and $\tau(x_U)$ be the (transformed) times to reach the critical degradation $\mathcal{D}_{\mathrm{f}}$ when the (transformed) accelerating variable take values x and x_U, respectively. Solving for $\tau(x)$ and τ_{x_U} the equation

$$\mathcal{D}_{\mathrm{f}} = \mathcal{D}[\tau(x), x, \boldsymbol{\beta}] = \mathcal{D}[\tau(x_U), x_U, \boldsymbol{\beta}]$$

gives

$$\frac{\tau(x_U)}{\tau(x)} = \frac{h_t[t(x_U)]}{h_t[t(x)]} = \exp[\beta_2(x - x_U)].$$

Using $\tau(x) = h_t[t(x)] = [t(x)]^{\kappa}$ and solving for $t(x_U)/t(x)$ yields

$$\mathcal{AF}(x) = \frac{t(x_U)}{t(x)} = \exp\left[\frac{\beta_2}{\kappa}(x - x_U)\right].$$

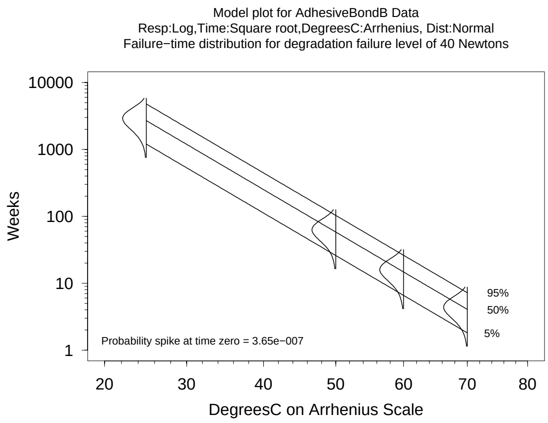

Figure 11: Adhesive Bond B data: model plot ML estimates of failure time distribution as function of temperature

7 Concluding Remarks and Extensions

Destructive degradation tests provide methods for assessing reliability even with few or no failures. The methodology described here can be extended in a number of different directions to handle various related problems that arise in practice. In particular,

- Our example had only one accelerating variable, which is the most common type of accelerated test. As explained in Section 2.3, however, the statistical extension to multiple accelerating variables is straightforward (and indeed has been implemented in the SPLIDA software, as described in Meeker and Escobar[4]. The most recent version of SPLIDA is always available at www.public.iastate.edu/˜splida). Possible interactions between accelerating variables can, however, complicate modeling and extrapolation to use conditions.
- Our modeling assumed that there is no measurement error. If there is a substantial amount of measurement error, then the estimate of σ will be biased high, providing similarly biased estimates of the failure-time distribution. If the magnitude of the measurement error standard deviation is known, the methods could be generalized to deal with this issue.
- In an extreme case, the degradation response may be given in terms of ordered categories (e.g., no degradation, light, medium, heavy, failed). Methods for handling such data are described in Agresti[1] and Johnson and Albert[2].
- In many applications, knowledge of the failure mechanism will provide useful prior information that can be used to improve, substantially, the

precision of estimates of life at use conditions. Bayesian methods can be used to handle such situations.

- The model used in this paper assumes that the relationship between transformed degradation and transformed time is linear at each level of temperature. It is easy to find examples in which this assumption will not hold (e.g., the models used in Meeker and LuValle[6]. The extension of the methods presented here to such nonlinear models is in principle, straightforward. There can, however, be identifiability and convergence problems in the ML methods.

- The predictions of reliability developed in this paper assume that the use environment is constant. Nelson[9, 11] describes statistical methods, based on a cumulative damage model, for estimating the life distribution under variable life conditions.

- Nelson[10] describes statistical methods for handling random initiation times that underlie some degradation processes.

- There are a number of open issues concerning the development of statistically efficient test planning that also meet practical constraints and provide useful amounts of power to detect departures fro the assumed model.

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